Brief Paper

Quadratic stabilizability of a new class of linear systems with structural independent time-varying uncertainty

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Abstract

This paper investigates the problem of designing a linear state feedback control to stabilize a new class of single-input uncertain linear dynamical systems. Uncertain parameters in the system matrices are time-varying and bounded in given compact sets. We first provide a concept called “new standard system”, where some of the entries are required to be negative sign-invariant and sign-invariant, and each entry varies independently in an arbitrarily large range. Then, for a class of new standard systems we derive a necessary and sufficient condition under which a system can be quadratically stabilized by a linear control for all admissible variations of uncertainties. The result extends the main result in Wei (1990. IEEE Transactions on Automatic Control, 35(3), 268–277). © 2000 Elsevier Science Ltd. All rights reserved.

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1. Introduction

In recent years, stabilizing an uncertain dynamical system has been a very active research area. See, for instance, the general linear matrix inequalities (LMIs) conditions by Boyd, El Ghaoui, Feron and Balakrishnan (1994) and some special geometrical structures by Yang and Zhang (1996), Hu, Dai, Jing and Zhang (1997), Wei (1990) and references therein. In this paper, we investigate uncertain dynamical systems whose parameters are time-varying and unknown but bounded in prescribed compact sets. We use a quadratic Lyapunov function to establish the stability of the closed-loop system. In principle, the quadratic stabilization problem can be solved by means of the LMI conditions proposed by Boyd et al. (1994), for which efficient algorithms are available. However, these conditions require to check a number of inequalities which is exponential in the number of uncertain parameters, so that the resulting number of LMIs cannot be handled unless the problem size is very small. Based on this trouble, it is necessary to supply some convenient ways to solve the quadratic stabilization problem.

Generally speaking, there are two categories of methods for quadratic stabilization. In the first method category, the uncertainties in system matrices usually allow to vary in sufficiently small ranges only and, therefore, are treated as perturbations (see e.g. Petersen & Hollot, 1986). Once the values of uncertainties are beyond the specified ranges, the systems may be no longer stabilizable. In the second method category, on the contrary, the system matrices may have some arbitrarily large varying terms. To guarantee robust stabilizability of an uncertain system, the uncertainties in the system matrices must be restricted such as the “matching conditions” (e.g., see Petersen, 1988), the “generalized matching conditions” (see Thorp & Barmish, 1981) and the “admissible-shuffle” structures (see Barmish, 1982) which are all sufficient conditions. Wei (1990) pointed out that all the stabilizable systems satisfying these sufficient conditions belong to some subsets of systems having a particular geometrical pattern called “antisymmetric stepwise (AS) configuration.” The “generalized AS configuration” was given by Wei (1989) as a sufficient condition on multi-input systems. The two particular geometrical patterns play a crucial role in ensuring robust...
stabilization of the systems. Thereafter, a number of authors have been paying attention to the direction in Wei (1990, 1989) and introducing new geometrical patterns, see for example the “generalized AS configuration” by Tsujino, Fujii and Wei (1993) as a necessary condition; the “strongly AS configuration” by Yang and Zhang (1996) and the “multi-input AS configuration” by Hu et al. (1997) as sufficient conditions. All these special geometrical patterns supply convenient ways to solve the quadratic stabilization problem. This paper falls into the second method category.

2. Preliminaries

Consider a linear time-varying uncertain system 
\[ \Sigma(A(q(t)), b(q(t))) \] (or uncertain system \( \Sigma(A(q), b(q)) \) for short) described by the state equation

\[
\dot{x}(t) = A(q(t))x(t) + b(q(t))u(t), \quad t \geq 0,
\]

where \( x(t) \in R^n \) is the state, \( u(t) \in R \) is the control, and \( q(t) \in R^n \) is the model uncertainty which is Lebesgue measurable, is restricted to a prescribed bounding set \( Q \) which is compact. Within this framework, \( A(\cdot) \) and \( b(\cdot) \) are \( n \times n \) and \( (n \times 1) \)-dimensional continuous functions on the set \( Q \), respectively. Hence, for fixed \( q \in Q \), \( A(q) \) and \( b(q) \) are the model matrices which result.

In this paper, unless otherwise stated, we assume that \( A(q) \) and \( b(q) \) depend on different components of \( q \); that is, we have \( q = [r:s] \), where \( A(\cdot) \) depends solely on \( r \) and \( b(\cdot) \) on \( s \). In the sequel, for notational simplicity, we always use \( \theta \) (or \( \bar{\theta} \)) to denote an entry which is a sign-invariant (or negative sign-invariant) uncertainty. Note that \( \theta \) (or \( \bar{\theta} \)) in different entries are not necessarily a same function of \( q \). \( I \) or \( I_n \) denotes the identity matrix, the norm of a real matrix \( M \triangleq [m_{ij}] \) will be taken to be the square root of the largest eigenvalues of \( M^T M \). Also \( \lambda_{\min/\max}[-] \) will denote the operation of taking the smallest (largest) eigenvalue. \( M(i:j) \) denotes the \( 2 \times 2 \) (or \( 1 \times 1 \) when \( i = j \) ) submatrix of an \( n \times n \) uncertain matrix \( M(q) \) defined by

\[
M(i:j) \triangleq \begin{bmatrix} m_{ii}(q) & m_{ij}(q) \\ m_{ji}(q) & m_{jj}(q) \end{bmatrix},
\]

where \( 1 \leq i \leq j \leq n \). The entry \( * \) in any matrix always denotes to be either zero or an uncertainty.

Definition 2.1. An uncertain system \( \Sigma(A(q), b(q)) \) is said to be quadratically stabilizable (QS) with respect to \( Q \) if there exist an \( n \times n \) positive-definite symmetric (or PDS for short) matrix \( P \), a positive constant \( \varepsilon \), and a continuous feedback control law \( u(\cdot): R^n \rightarrow R \) with \( u(0) = 0 \) satisfying the following conditions. Given any admissible uncertainty \( q(\cdot) \), it follows that

\[
L(x, t) \triangleq x^T [A^T(q(t))P + PA(q(t))]x + 2x^TPb(q(t))u(x) \leq -\varepsilon \|x\|^2
\]

for all pairs \( (x, t) \in R^n \times [0, +\infty) \). \( L(x, t) \) is the so-called Lyapunov derivative associated with the quadratic Lyapunov function \( V(x) = x^T P x \). Furthermore, \( \Sigma(A(q), b(q)) \) is said to be quadratically stabilizable via linear control (QS VLC) with respect to \( Q \) if \( u(x) = Kx \) where \( K \) is an \( n \) constant row vector.

Definition 2.2. An \( (n + 1) \times (n + 1) \) uncertain matrix \( M(q) \triangleq [m_{ij}(q)] \) is said to be in the new standard form if there exist two integers \( i^* \) and \( j^* \) satisfying \( 0 \leq i^* \leq n \), \( 0 \leq j^* \leq n \) and \( 1 \leq i^* + j^* \leq n \) such that \( m_{ii}(q)(1 \leq i \leq i^* + j^*) \) and \( m_{i+1,i}(q)(i^* + 1 \leq i \leq n) \) are independent negative sign-invariant and sign-invariant functions of \( q \) (including a constant function), respectively.

In this paper, for space reason we always assume \( i^* = 0 \) and, unless otherwise stated, \( 1 \leq j^* \leq n \). The other cases are discussed in our internal laboratory report. Therefore, for example, when \( j^* = 2 \) and \( n = 4 \) the new standard form can be described as:

\[
\begin{bmatrix}
\emptyset & \emptyset & * & * \\
* & \emptyset & * & * \\
* & * & * & * \\
* & * & * & * \\
\end{bmatrix}
\]

Definition 2.3. An uncertain system \( \Sigma(A(q), b(q)) \) is said to be in the new standard form with structural independent uncertainties if its corresponding square matrix \( M(q) \) defined as

\[
M(q) = \begin{bmatrix} A(q) & b(q) \\ 0 & 0 \end{bmatrix} \triangleq [m_{ij}(q)]
\]

is in the new standard form and every entry \( m_{ij}(q) \) (except for \( m_{ii}(q)(1 \leq i \leq n) \) and \( m_{i+1,i}(q)(1 \leq i \leq n) \)) is zero or an uncertainty varying independently in \( [-r_{ij} r_{ij}] \) where \( r_{ij} > 0 \) may be arbitrarily large.

Remark 2.4. Obviously, the new standard form is different from the standard form in Wei (1990). If \( j^* \) may be permitted to be zero, the new standard form may be viewed as an extension of the standard form in Definition 2.2 by Wei (1990) and, thus, the former is of more practical sense than the latter.
Lemma 2.5 (see Barmish (1985) for proof). An uncertain system $\Sigma(A(q), b(q))$ is QS $\iff$ there exists an $n \times n$ PDS matrix $S$ such that $x^T(A(q)S + SA^T(q))x < 0$ for all pairs $(x, q) \in N \times Q$ with $x \neq 0$ where $N \triangleq \{x \in \mathbb{R}^n : b^Tx = 0 \text{ for some } b \in \text{conv}\{b(q): q \in Q\}\}$.

Corollary 2.5.1 (Wei, 1990, Corollary 4.2). An uncertain system $\Sigma(A(q), b(q))$ with $b(q) = [0 \cdots 0 \ 0]^T$ is QS $\iff$ there exists an $n \times n$ PDS matrix $S$ such that

$$\pi(S, q) \triangleq \Theta^T(A(q)S + SA^T(q))$$

is negative definite symmetric (or NDS for short) for all $q \in Q$ where $\Theta = [I_{n-1} : 0]^T$ is $n \times (n-1)$ matrix.

The pair $(S, \pi)$ satisfying Corollary 2.5.1 is called an admissible pair for $\Sigma(A(q), b(q))$.

Definition 2.6. Consider an uncertain system $\Sigma(A(q), b(q))$. When $b(q) = [0 \cdots 0 \ 0]^T$, a second down-augmented system $\Sigma^+(A^-(q), b^-(q))$ of $\Sigma(A(q), b(q))$ is defined as follows:

$$A^+(q) \triangleq \begin{bmatrix} A(q) & b(q) \\ \ast & \ast & \cdots & \ast \\ \ast & \ast & \cdots & \ast \\ \ast & \ast & \cdots & \ast \end{bmatrix} = \begin{bmatrix} A(q) & b(q) \\ c(q) & a_{n+1 \times n+1} \\ \ast & \ast & \cdots & \ast \end{bmatrix}.$$  

$$b^+(q) \triangleq [0 \cdots 0 \ 0]^T.$$  

If $a_{n+1 \times n+1} = \emptyset$, then $\Sigma^+(A^-(q), b^-(q))$ is called a first down-augmented system of $\Sigma(A(q), b(q))$. For convenience, we call the first (or second) down-augmented system, a down-augmented system.

Lemma 2.7 (see the appendix for proof). Consider uncertain systems $\Sigma(A(q), b(q))$ and $\Sigma^+(A^-(q), b^-(q))$ as in Definition 2.6. Then, $\Sigma(A(q), b(q))$ is QS VLC $\iff$ $\Sigma^+(A^-(q), b^-(q))$ is also QS VLC.

Remark 2.8. Lemma 3.6 in Wei (1990) implies $\Sigma^+(A^-(q), b^-(q))$ is QS only if $\Sigma(A(q), b(q))$ is QS where $b(q) = [0 \cdots 0 \ 0]^T$. In fact, Lemma 2.7 shows that $\Sigma^+(A^-(q), b^-(q))$ and $\Sigma(A(q), b(q))$ are equivalent as far as QS VLC is concerned. Moreover, Lemma 2.7 may be considered to be an extension of Theorem 3.1 in Barmish (1983) under the case when $m = 1$.

Definition 2.9. Consider an uncertain system $\Sigma(A(q), b(q))$ with $b(q) = [0 \cdots 0 \ 0]^T$. A second up-augmented system $\Sigma^+(A^+(q), b^+(q))$ of $\Sigma(A(q), b(q))$ is defined as follows:

$$A^+(q) \triangleq \begin{bmatrix} \emptyset \ \ast \ \cdots \ \ast \\ 0 & A & \ast \ast \ast \\ \ast & \ast & \cdots & \ast \\ \ast & \ast & \cdots & \ast \end{bmatrix} = \begin{bmatrix} \emptyset \ \ast \ \cdots \ \ast \\ 0 & A & \ast \\ a_{n0} & \ast & \cdots & \ast \end{bmatrix},$$  

$$b^+(q) = \begin{bmatrix} 0 \\ b(q) \end{bmatrix}.$$  

For convenience, we call a first (or second) up-augmented system an up-augmented system.

Lemma 2.10 (see the appendix for proof). Consider uncertain systems $\Sigma(A(q), b(q))$ and $\Sigma^+(A^+(q), b^+(q))$ as in Definition 2.9. Then, $\Sigma(A(q), b(q))$ is QS VLC $\iff$ $\Sigma^+(A^+(q), b^+(q))$ is also QS VLC.

Definition 2.11 (Wei, 1990, Definition 4.7). Consider an uncertain system $\Sigma(A(q), b(q))$ with

$$A(q) = \begin{bmatrix} 0 & A^{-}(q) \\ \ast & \ast \cdots \ast \end{bmatrix}, \ b(q) = \begin{bmatrix} 0 \\ \emptyset \end{bmatrix}.$$  

A first up-augmented system $\Sigma^+(A^+(q), b^+(q))$ of $\Sigma(A(q), b(q))$ is defined as follows:

$$A^+(q) = \begin{bmatrix} 0 & 0 & \cdots & \ast \\ 0 & 0 & \cdots & \ast \\ \ast & \ast & \cdots & \ast \end{bmatrix},$$  

$$b^+(q) = \begin{bmatrix} 0 \\ b(q) \end{bmatrix}.$$  

For convenience, we call a first (or second) up-augmented system an up-augmented system.

Lemma 2.12. Consider uncertain systems $\Sigma(A(q), b(q))$ and $\Sigma^+(A^+(q), b^+(q))$ as in Definition 2.11. Then, $\Sigma(A(q), b(q))$ is QS VLC $\iff$ $\Sigma^+(A^+(q), b^+(q))$ is QS VLC.

Noting Lemma A.2 and the proof procedure of Lemma 2.10, we can similarly prove Lemma 2.12.

3. The main result

In this section, we first supply some necessary definitions and lemmas. Then we give our main result.

Definition 3.1. An uncertain system $\Sigma(A(q), b(q))$ is said to have a new antisymmetric stepwise configuration (or NAS configuration for short), if $M(q)$ as in (3) satisfies the following conditions.

1. $m_{ii} (1 \leq i \leq j^ **) \text{ and } m_{i+1} (1 \leq i \leq n) \text{ are negative sign-invariant and sign-invariant, respectively.}$
2. If $p \geq k + 2, 1 \leq k \leq j^ * \text{ and } m_{kk}(q) \neq 0$, then $m_{uv}(q) \equiv 0$ for all $u > v, u \neq p - 1 \text{ and } v < k$.
3. If $p \geq k + 2, j^ * < k \leq n - 1 \text{ and } m_{uv}(q) \neq 0$, then $m_{uv}(q) \equiv 0$ for all $u \geq v, j^ * < u \leq p - 1, j^ * \leq v < k $ and for all $u > v, 1 \leq v \leq j^ *, u \leq p - 1$ where $j^ * \geq 1$.  


Example 3.2. For $4 \times 4$ matrices (where $j^p = 2$) all possible NAS configurations are as follows:

\[
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
0 & 0 & * & 0 \\
0 & 0 & 0 & 0 \\
* & * & * & * \\
* & * & * & *
\end{bmatrix}
\begin{bmatrix}
0 & 0 & * & * \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
0 & 0 & * & * \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
0 & 0 & * & * \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
0 & 0 & * & * \\
0 & 0 & 0 & 0 \\
* & * & * & * \\
* & * & * & *
\end{bmatrix}
\begin{bmatrix}
0 & 0 & * & * \\
0 & 0 & 0 & 0 \\
* & * & * & * \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
0 & 0 & * & * \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
0 & 0 & * & * \\
0 & 0 & 0 & 0 \\
* & * & * & * \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
0 & 0 & * & * \\
0 & 0 & 0 & 0 \\
* & * & * & * \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
0 & 0 & * & * \\
0 & 0 & 0 & 0 \\
* & * & * & * \\
* & * & * & *
\end{bmatrix}
\]

Remark 3.3. (i) Comparing Example 3.2 above with Example 2.8 in Wei (1990) we easily see that Definition 3.1 is different from Definition 2.5 in Wei (1990). (ii) If $j^p = 0$, we also see that Definition 3.1 is just as defined in Definition 2.5 in Wei (1990) which is thus extended to a more general case. (iii) Ishida, Adachi and Tokumaru (1981) studied robust stability of an uncertain system (without control) being composed of 0, positive and negative sign-invariant entries. In Definition 3.1, we easily see that an uncertain system $\Sigma(A(q),b(q))$ having an NAS configuration is only composed of 0, *, positive and negative sign-invariant entries.

Comparing Definitions 2.6, 2.9 and 2.11 with Definition 3.1, we find an important fact.

Fact 3.4. If an uncertain system $\Sigma(A(q),b(q))$ satisfies the following condition: $M(q)$ as in (3) has an NAS configuration. Then $\Sigma^+(M(q),b^+)$ with $b^+ = [0 \cdots 0 \ 1]^T$ can always be generated from the simplest system $\Sigma(A_0(q),b_0(q))$, where $A_0(q) = [\ast]$ (or $[\theta]$), $[0]$ and $b_0(q) = [\theta]$, via a sequence of augmentations (either down or up). Furthermore, once we take a second up-augmented (or second down-augmented) operation, we can no longer take a first up-augmented (or first down-augmented) operation.

This fact reveals a secret: an NAS configuration is just constructed by a sequence of augmentations. Fact 3.4 can be viewed as an alternative definition of NAS configuration and will play a key role in proving the sufficiency part of Theorem 3.9. Next, lemmas will be used to prove the necessity part of Theorem 3.9.

Lemma 3.5. Consider the free system $\Sigma(A_i,b_i)$ where $b_i = [0 \cdots 0 \ 1]^T$ and $A_i = (a_{ij})_{n \times n}$ satisfies the following conditions: $a_{ij} = -1 (i = 1, \ldots, j^p)$, $a_{i+1,j} = 1 (i = 1, \ldots, n-1)$ and the other entries are all zero where $0 \leq j^p \leq n$. If there is an admissible pair $(S,\pi)$ for $\Sigma(A_i,b_i)$, then some entries of $S$ have the following properties: (1) $s_{ii} > 0$ for all $i = 1, \ldots, n$ and $s_{ii}s_{jj} > s_{ij}^2$ for all $i,j = 1, \ldots, n$ and $i \neq j$; (2) If there exists some $i \leq j \leq n-1$ such that $s_{i+1,i+1} < r^{2(n-1)}s_{ii}$, then for any integer $k$ satisfying $i-1 \leq k \leq 1$, $s_{i+1,k+1} < r^{2(n-1)}s_{kk}$ and $|s_{k+1,i}| < r^{2(n-1)}s_{kk}$; and (3) If there exists some $i \leq j \leq n-1$, then for any integer $k$ satisfying $i+1 \leq k \leq n-1$, $s_{i+1,k+1} < r^{2(n-1)}s_{kk}$ and $s_{k+1,i+k+1} > r^{2(n-1)}s_{kk}$ where $r$ is greater than one and may be large enough.

Lemma 3.6. Consider an uncertain system $\Sigma(A_i,b_i)$ where $b_i = [0 \cdots 0 \ 1]^T$ and $A_i = (a_{ij})_{n \times n}$ satisfies the following conditions: (i) $a_{ii} = -1 (i = 1, \ldots, j^p)$ and $a_{i+1,i} = 1 (i = 1, \ldots, n-1)$; (ii) There is a structural uncertainty $a_{uv}$ where $1 \leq u \leq v$ and $v < u \leq n-1$ (or $j^p < v$ and $v \leq u \leq n-1$), i.e., $|a_{uv}| \leq \bar{a}_{uv}$ and $\bar{a}_{uv}$ may be arbitrarily large; and (iii) There are all entries zero except for $a_{ii}, a_{i+1,i}$ and $a_{uv}$, where $0 \leq j^p \leq n$. If there is an admissible pair $(S,\pi)$ for $\Sigma(A_i,b_i)$, then the entries $s_{uu}, s_{u+1,u}$ and $s_{u+1,u+1}$ of $S$ satisfy

\[
s_{u+1,u+1} > r^{2(n-u)}s_{uu} \quad \text{and} \quad s_{u+1,u+1} > r^{2(n-u)}s_{uu+1}, \tag{5}
\]

where $1 < r < \bar{a}_{uv}$ and $r$ may be large enough.

Lemma 3.7. Consider an uncertain system $\Sigma(A_i,b_i)$ where $A_i$ is just as in Lemma 3.5 and $b_i = [0 \cdots 0 \ a_{kp} \ 0 \cdots 0 \ 1]^T$ where $1 \leq k \leq n-1$, $a_{kp}$ is a structural uncertainty, i.e., $|a_{kp}| \leq \bar{a}_{kp}$ and $\bar{a}_{kp}$ may be arbitrarily large, and $0 \leq j^p \leq n$. If there is an admissible pair $(S,\pi)$ for $\Sigma(A_i,b_i)$, then the entries $s_{kk}, s_{k+1,k}$ and $s_{k+1,k+1}$ of $S$ satisfy

\[
s_{k+1,k+1} > r^{2(n-k)}s_{kk} \quad \text{and} \quad |s_{k+1,k}| < r^{2(n-k)}s_{kk}, \tag{6}
\]

where $1 < r < \bar{a}_{kp}$ and $r$ may be large enough.

Lemma 3.8. Consider an uncertain system $\Sigma(A_i,b_i)$ where $b_i(q)$ is just as in Lemma 3.7 and $A_i = (a_{ij})_{n \times n}$ satisfies the following conditions: (i) $a_{ii} = -1 (i = 1, \ldots, j^p)$ and $a_{i+1,i} = 1 (i = 1, \ldots, n-1)$ where $1 \leq j^p \leq n$; (ii) $a_{a1}$ is a structural uncertainty, i.e., $|a_{a1}| \leq \bar{a}_{a1}$ and $\bar{a}_{a1}$ may be arbitrarily large, and $a_{a1}$ is independent from $a_{kp}$; and (iii) The other entries are all zero except for $a_{ii}, a_{i+1,i}$ and $a_{kp}$ and $a_{a1}$. Then the system is not QS.

The proofs of Lemmas 3.5–3.8 can be found in the appendix. We now state our main result.

Theorem 3.9. A system $\Sigma(A,q,b(q))$ in the new standard form (where $i^p = 0$ and $j^p = 1$) with structural independent uncertainties as in Definition 2.3 is QSVL if and only if it has a NAS configuration.
A_0(q) = [\emptyset] and b_0(q) = \emptyset; or, A_0(q) = [0] and b_0(q) = 0, via a sequence of augmentations (either down or up). It is trivial that \( \Sigma(A_0(q), b_0(q)) \) is QSVLC. Following the augmentation order and applying Lemmas 2.7, 2.10 and 2.12 repeatedly, we can derive \( \Sigma^+(M(q), b^+) \) is QSVLC and so is \( \Sigma(A(q), b(q)) \).

(Necessity). For notational simplicity, we prove the necessity part for a special case when all sign-invariant (or all negative sign-invariant) uncertain entries of \( \Sigma(A(q), b(q)) \) are identical. However, with slight modification, the proof is also valid for the general cases.

According to the definition of NAS configuration, it suffices to show that if a free system \( \Sigma(A_i, b_i) \) as in Lemma 3.5 has two independent structural uncertainties \( a_{ii} \) and \( a_{np} \) with \( u > v, p \geq k + 2, 1 \leq k \leq j^p, \) \( v \leq k \) and \( u \leq p - 1, \) or \( u > v, j^p < v \leq k + 1, p \geq k + 2, j^p < k \leq n - 1 \), or \( j^p < u \leq p - 1, \) or \( u > v, 1 \leq v \leq j^p, p \geq k + 2, j^p < k \leq n - 1 \) and \( u \leq p - 1, \) then the system is not QS.

Proceeding by contradiction. Suppose the system is QS. Note Lemmas 2.7, 2.10, \( j^p > 0, \) and Theorem 3.2 in Wei (1990), we only need to consider the system \( \Sigma(A^q(q), b^q(q)) \) is QS where \( b^q(q) = [0 \cdots 0 a_{nn+1} \cdot \cdots 1] \) (1 \( \leq k \leq n - 1 \)) and \( A^q(q) \) satisfying the following conditions: (i) \( a_{ii} = -1 \) (i = 1, \ldots, \( j^p \)) and \( a_{i+1} = 1 \) (i = 1, \ldots, n - 1) where 1 \( \leq j^p \leq n; \) (ii) \( a_{ii} \) (2 \( \leq i \leq n \)) is a structural uncertainty and independent from \( a_{np+1} \); and (iii) The other entries are all zero except for \( a_{ii}, a_{i+1}, a_{n+1} \) and \( a_{ii}. \)

In the following, we assume \( j^p = n \) and consider \( \Sigma(A^q(q), b^q(q)) \) according to three cases. As to the cases when 1 \( \leq j^p \leq n - 1 \), we can derive the contradiction in a similar method.

Case 1 : \( u = n \) and 1 \( \leq k \leq n - 1 \). Lemma 3.8 shows that the system cannot be QS.

Case 2 : \( 2 \leq u \leq k \) and 2 \( \leq k \leq n - 1 \). Since \( a_{uu} \) is a structural uncertainty, it follows from Lemma 3.6 that
\[
s_{u+1} + 1 + u > r^{2(n-u)}s_{uu} \text{ where } 1 < r < a_{uu}. \]
According to \( a_{np} \) which is a structural uncertainty, it follows from Lemma 3.7 that \( s_{k+1} + 1 < r^{2k-n}a_{kk} \) where 1 \( < r < a_{kk}. \) Note property (2) in Lemma 3.5 and \( u \leq k \) we immediately obtain
\[
s_{u+1} + 1 + u < r^{2(n-u)}s_{uu} \text{ which contradicts } s_{u+1} + 1 + u > r^{2(n-u)}s_{uu}. \]

Case 3 : \( k + 1 \leq u \leq n - 1 \) and 1 \( \leq k \leq n - 2 \). It follows from Lemma 2.5 that there exists a PDS matrix \( S \) such that \( x^T(A(q)S + S^T A(q)x) < 0 \) for all pairs \((x, q) \in N \times Q \) with \( x \neq 0 \) where \( N = \{x \in R^n : b^T x = 0 \text{ for some } b \in \text{ conv} (b(q) \in Q)\}. \) Applying the same method as in the proof of Lemma 4.12 in Wei (1990) yields an equivalent condition \( y^T \Sigma(q)y < 0 \) for all \( q \in Q \) and \( y(1 \neq 0) \in R^{n-1} \) where \( i \) (when \( u = n - 1 \)) the part entries of \( \Sigma(q), \) are as follows:
\[
p_{kk} = 2s_{k+1} + 1 - s_{kk} + a_{k+1}(-2s_{k+1} + s_{k+1} + 1 + s_{k+1} + 1 + s_{k+1} = -2s_{k+1} + s_{k+1} + 1 + s_{k+1} + 1 + s_{k+1} + 1 = 2(s_{k+1} - s_{k+1} - n - 1), \text{ and } p_{uu} = 2s_{u+1} + 1 - s_{uu} + a_{u+1} s_{uu} \text{ at } u = n - 1, \) the part entries of \( \pi(S, q) \) are as follows: \( p_{kk} \) and \( p_{11} \) as are in (i) above, \( p_{k+1} = a_{k+1} - s_{k+1} - s_{k+1} - s_{k+1} + s_{k+1} + a_{k+1}(-2s_{k+1} + s_{k+1} + 1 + s_{k+1} + 1 = 2(s_{k+1} - s_{k+1} - n - 1), \text{ and } p_{u+1} = 2s_{u+1} + 1 - s_{uu} + a_{u+1} s_{uu} \text{ at } u = n - 1. \)

Case 1 : \( u = n - 1 \). It follows from the negative definiteness of \( \Sigma(q), \) that \( \pi(k : n - 1) > 0 \) and \( \pi(1u) > 0; \) i.e.,
\[
4(s_{u+1} - s_{u+1} - n - 1)(s_{u+1} - s_{u+1} - n - 1) + a_{u+1}(-2s_{u+1} + s_{u+1} + 1) > 0.
\]
(7) and
\[
4(s_{u+1} - s_{u+1} - n - 1)(s_{u+1} - s_{u+1} - n - 1) + a_{u+1}(-2s_{u+1} + s_{u+1} + 1) > 0.
\]
(8)
Replacing \( a_{u+1} \) in (7) by \( -a_{u+1} \), we get a new inequality. Adding this new inequality and (7) yields
\[
4(s_{u+1} - s_{u+1} - n - 1)(s_{u+1} - s_{u+1} - n - 1) > 0.
\]
(9)
Similarly, from (8) we may deduce
\[
4(s_{u+1} - s_{u+1} - n - 1)(s_{u+1} - s_{u+1} - n - 1) > 0.
\]
(10)
When \( a_{u+1} \equiv 0, \) from Lemma 3.6, (5) holds; When \( a_{u+1} \equiv 0, \) from Lemma 3.7, (6) holds. Next, assume \( s_{uu} > r^{n-k+2}s_{k+1} \). According to (5), property (3) of Lemma 3.5 and \( u \leq n - 2 \), we immediately have
\[
s_{uu} > r^{2n-u} s_{uu} \text{ and } s_{uu} > r^{2n-u} s_{uu}. \]
(11)
Set \( a_{u+1} = r, \) (9) becomes
\[
4(s_{u+1} - s_{u+1} - n - 1)(s_{u+1} - s_{u+1} - n - 1) > 0.
\]
(12)
Then, when \( r \) is large enough, combining \( s_{uu} > r^{n-k+2}s_{k+1} \), (11) and (6) with (12), we easily draw \( 8/7 s_{uu} > r^{2n-u} s_{uu}. \)
(13)
which is a contradictory requirement if \( r \) is large enough. Hence, \( s_{uu} \leq r^{n-k+2}s_{k+1} \). It follows from property (2) of Lemma 3.5, (6) and \( s_{uu} \leq r^{n-k+2}s_{k+1} \) that
\[
s_{uu} < r^{n-k+2}2^{2(n-k)+1}2^{n-k+1} \ldots 2^{2(n-u-1)}s_{uu} \leq r^{n-k+2}s_{k+1}.
\]
(14)
On the other hand, from (6) and property (2) of Lemma 3.5, we have \( |S_{21}| < r^{n-k+2}s_{k+1} \). Then, when \( r \) is large enough, combining (10), (5) and \( |S_{21}| < r^{n-k+2}s_{k+1} \) yields
\[
8r^{2n-u} s_{uu} \geq 2s_{u+1} + 1 > 0.
\]
(15)
It follows from property (3) of Lemma 3.5 and (5) that
\[ s_m > t^{2u_1} t^{2} \cdots t^{2(n-u_1)-1} s_{u_1+1} u_1+1 \]
\[ = \frac{1}{16} t^{2u_1} t^{2} \cdots t^{2(n-u_1)-1} s_{u_1+1} u_1+1. \]  \quad (16)
Combining (15) and (16) leads to
\[ s_m > t^{2u_1} t^{2} \cdots t^{2(n-u_1)-1} s_{u_1+1} u_1+1, \]  \quad (17)
Since \( \delta a_{u_1} \) may be arbitrarily large, we may select \( \delta a_{u_1} < \delta a_{u_1} \) to guarantee \( \frac{1}{16} s_{u_1+1} > \frac{1}{16} r^{2u_1} \). Then, it follows from (17) and \( \frac{1}{16} r^{2u_1} > \frac{1}{16} s_{u_1+1} ) \) which contradicts (14).

The proof under Case ii is analogous to the proof under Case i and hence omitted. The proof is completed. \( \square \)

\textbf{Remark 3.10.} (i) From the proof above, we easily know that Lemmas 3.5–3.7 play a key role. In fact, we can also give a simpler proof of Theorem 3.2 (Necessity) in Wei (1990) by applying Lemmas 3.5–3.7. The detailed proof is omitted here. (ii) When \( j^* \) in Theorem 3.9 may be 0, Theorem 3.9 is Theorem 3.2 by Wei (1990). Hence, Theorem 3.9 extends Theorem 3.2 in Wei (1990). (iii) Noting the proofs of Lemmas 2.7, 2.10 and 2.12 and Theorem 3.9, we can easily design a desired linear controller for an uncertain system having an NAS configuration and the procedure is omitted here.

Note Corollaries 3.3 and 3.4 in Wei (1990), we similarly have some related results omitted here.

\section{Conclusion}

In this paper, we studied the quadratic stabilizability of single-input linear systems with structural independent time-varying uncertainties. By introducing a concept called “new standard form”, we have derived a necessary and sufficient condition under which the system is QSVLC. Based on the condition, to judge if a system can be QSVLC, we only need to check if all uncertainties in the system matrices form a special geometrical pattern called NAS configuration. Our result extends the main result in Wei (1990).

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\section*{Appendix}

In this appendix, some related knowledge can be found in the proof of Theorem 3.1 in Barmish (1983).

\textbf{Proof of Lemma 2.7 (Necessity).} Suppose that \( \Sigma(A(q), b(q)) \) is QSVLC, that is, there exists a linear stabilizing control \( u = Kx \) for \( \Sigma(A(q), b(q)) \) and an \( n \times n \) PDS matrix \( S \) such that
\[ (A(q) + b(q)K)S + S(A(q) + b(q)K)^T = \tilde{A}(q) + S \tilde{A}^T(q) \]  \quad (A.1)
is NDS for all \( q \in Q \). We call \((S, K)\) a desired pair for \( \Sigma(A(q), b(q)) \). Our task is to construct a desired pair \((S^+, K^+)\) for \( \Sigma^+(A^+(q), b^+(q)) \) such that
\[ (A^+(q) + b^+(q)K^+)S^+ + S^+(A^+(q) + b^+(q)K^+)^T \]
\[ = \tilde{A}^+(q)S^+ + S^+ \tilde{A}^{+T}(q) \]  \quad (A.2)
is NDS for all \( q \in Q \). Partition \( S^+ \) and \( K^+ \), respectively, as
\[ S^+ \triangleq \begin{bmatrix} s_{11}^+ & s_{12}^+ \\ s_{12}^+ & s_{22}^+ \end{bmatrix} \quad \text{and} \quad K^+ \triangleq [K_{1}^+ k_{n+1}^+] \]  \quad (A.3)
with \( \dim s_{11}^+ = n \times n, \dim s_{12}^+ = n \times 1, \dim s_{22}^+ = 1 \times 1 = \dim k_{n+1}^+ \) and \( \dim K_1^+ = 1 \times n \).

Select \( s_{11}^+ = S, s_{12}^+ = SK^T, \) and \( s_{22}^+ = r \), where \( r > 0 \) is chosen sufficiently large so that \( S^+ \) is PDS and \( r > KSK^T \). Select \( K_1^+ = -k_{n+1}^+K \). Then compute \( \tilde{A}^+(q)S^+ + S^+ \tilde{A}^{+T}(q) \). We can easily choose a suitable \( k_{n+1}^+ \) to guarantee that \( \tilde{A}^+(q)S^+ + S^+ \tilde{A}^{+T}(q) \) is NDS for all \( q \in Q \).

From selection of \( S, K \) in the proof of Theorem 3.1 in Barmish (1983) and negative definiteness of \( \tilde{A}^+(q)S^+ + S^+ \tilde{A}^{+T}(q) \), we can easily prove sufficiency part of Lemma 2.7. \( \square \)

To prove Lemma 2.10, we first introduce the following lemma.

\textbf{Lemma A.1.} Consider an uncertain system \( \Sigma(A(q), b(q)) \) with \( A(q) = \{a_i\}_{n \times n} \) and \( b(q) = [0 \cdots 0 \ 0]^T \). Its up-augmented system \( \Sigma^+(A^+(q), b^+(q)) \) is defined as follows:
\[ A^+(q) = \begin{bmatrix} 0 & a_{01} & \cdots & a_{0n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & A(q) \end{bmatrix} \]
\[ b^+(q) = \begin{bmatrix} 0 \\ b(q) \end{bmatrix} \]  \quad (A.4)
Then \( \Sigma(A(q), b(q)) \) is QSVLC \( \iff \Sigma^+(A^+(q), b^+(q)) \) is QSVLC.

\textbf{Proof of Lemma A.1 (Necessity).} Suppose that \( \Sigma(A(q), b(q)) \) is QSVLC, that is, there exists a desired pair \((S, K)\) such that (A.1) is NDS for all \( q \in Q \). Now our object
is to select a desired pair \((S^+, K^+)\) to guarantee (A.2) is NDS for all \(q \in Q\). To accomplish this, we partition \(S^*\) and \(K^*\) as

\[
S^+ = \begin{bmatrix} s_{00}^+ & s_{01}^+ \\ s_{01}^+ \end{bmatrix} \quad \text{and} \quad K^+ = [k_0^+ \quad K_1^+] \tag{A.5}
\]

with \(\dim s_{00}^+ = 1 \times 1 = \dim k_0^+\), \(\dim s_{01}^+ = 1 \times n = \dim K_1^+\), \(s_{01}^+ = n \times n\).

Select \(s_{01}^+ = 0\), \(s_{00}^+ = r > 0\) (to guarantee \(S^+\) is PDS), \(s_{11}^+ = S, \ k_0^+ = 1/r\), and \(K_1^+ = K\). Compute \(\bar{A}(q)S^+ + S^+\bar{A}^T(q)\). We can choose a suitable \(r > 0\) to guarantee \(\bar{A}(q)S^+ + S^+\bar{A}^T(q)\) is NDS \(\forall q \in Q\).

(Sufficiency). Assume \(\Sigma^+(A^+(q), b^+(q))\) is QSVC; i.e., there exists a desired pair \((S^*, K^*)\) as in (A.5) to ensure (A.2) is NDS \(\forall q \in Q\). Choosing \(S = s_{11}^+\) and \(K = K_1^+ + k_0 s_{01}s_{11}^{-1}\) can ensure (18) is NDS \(\forall q \in Q\).

**Proof of Lemma 2.10** (*Necessity*). Suppose that \(\Sigma(A(q), b(q))\) is QSVC. Set

\[
A(q) = \begin{bmatrix} A_1(q) \\ A_2(q) \end{bmatrix}
\]

Consider its up-augmented system \(\Sigma(A_1(q), b_1(q))\) with

\[
A_1(q) = \begin{bmatrix} \bar{a} & \ast & \ast & \ast \\ 0 & \bar{a}(q) \\ 0 & A_2(q) & \bar{b}(q) \end{bmatrix}
\]

\[
b_1(q) = \begin{bmatrix} 0 \\ b(q) \end{bmatrix}
\]

From Lemma A.1 we see \(\Sigma(A_1(q), b_1(q))\) is QSVC. From \(A_1(q)\) construct a system \(\Sigma(A_2(q), b_2(q))\) with

\[
A_2(q) = \begin{bmatrix} \bar{a} & \ast & \ast \\ 0 & A_2(q) \end{bmatrix}
\]

\[
b_2(q) = \begin{bmatrix} \ast \\ \bar{b}(q) \end{bmatrix}
\]

Compare \(\Sigma(A_2(q), b_2(q))\) and \(\Sigma(A_1(q), b_1(q))\), we easily find \(\Sigma(A_1(q), b_1(q))\) is a down-augmented operation of \(\Sigma(A_2(q), b_2(q))\) as in Definition 2.6. Hence, it follows from Lemma 2.7 that \(\Sigma(A_2(q), b_2(q))\) is QSVC. Similarly, it is seen that \(\Sigma^+(A^+(q), b^+(q))\) is a down-augmented operation of \(\Sigma(A_2(q), b_2(q))\). Then, from Lemma 2.7 it follows that \(\Sigma^+(A^+(q), b^+(q))\) is QSVC.

(Sufficiency). Suppose that \(\Sigma^+(A(q), b^+(q))\) is QSVC. Noting \(a_{00} \in A^+(q)\) is a structural independent uncertainty. Set \(a_{00} = 0\), from Lemma A.1 we immediately have \(\Sigma(A(q), b(q))\) is QSVC.

The following lemma is useful for proving Lemma 2.12.

**Lemma A.2.** Consider uncertain systems \(\Sigma(A(q), b(q))\) and \(\Sigma^+(A^+(q), b^+(q))\) as in Definition 2.11 where \(a_{00} \equiv 0\). Then \(\Sigma(A(q), b(q))\) is QSVC \(\Leftrightarrow \Sigma^+(A^+(q), b^+(q))\) is QSVC.

**Proof of Lemma A.2** (*Necessity*). Repeating the procedure in the proof (*Necessity*) of Lemma A.1, we easily choose a desired pair \((S^+, K^+)\) for \(\Sigma^+(A(q), b^+(q))\). For instance, we select \(s_{00}^+ = r > 0\), \(s_{11}^+ = S, \ k_0^+ = 1/r\), and \(K_1^+ = K = [k_1 k_2 \cdots k_n]\), and \(s_{01}^+ \text{ and } k_0^+\) satisfy the following conditions: (i) When \(\theta\) is positive sign-invariant, \(s_{01}^+ = [-r, 0, \ldots , 0]^T\) and \(k_0^+ = k_1/r^2\); (ii) When \(\theta\) is negative sign-invariant, \(s_{01}^+ = [r, 0, \ldots , 0]^T\) and \(k_0^+ = -k_1/r^2\) where \(r\) is large enough. The proof (Sufficiency) of Lemma A.2 is similar to that of Lemma A.1. □

Next, in the proofs of Lemmas 3.5–3.8, we consider \(j^* = n\). The reasons are similar for \(0 \leq j^* < n\).

**Proof of Lemma 3.5.** Recalling that \(\pi(S) = \Theta^T(A, S + SA_0^T)\Theta\) where \(\Theta = [I_{n+1}, 0]^T\), the entries \(\pi_{ij}\) of \(\pi(S)\) are \(\pi_{ij} = -2s_{ij} + s_{i+1,j} + s_{i,j+1}\). Property (1) immediately follows from the positive definiteness of \(S\). We now prove property (2). In view of the hypothesis that \(s_{i+1} \geq r^{2(n-i)+1}s_{ii}\) we first prove that \(s_{ii} < r^{2(n-i)+1}s_{ii}\). From property (3) is entirely the same as above, we omit it for the sake of brevity. □

**Proof of Lemma 3.6.** A straightforward computation of \(\pi(S, q)\) yields \(\pi_{uu} = 2(s_{u+1}-s_{uu} + a_{uu} s_{uu})\), \(\pi_{uv} = (S, q)\) yields \(\pi_{uv} = (A.7)\).


\[ 2(s_{v+1} - s_{e}) \text{ and } \sigma_{uv} = -2s_{uv} + s_{nu} + s_{nu+1} + a_{uw} s_{wv}. \]

From \( \det(\pi(u, v)) > 0 \) we have

\[ 4(s_{v+1} - s_{e})(s_{n+1} - s_{m} + a_{uw} s_{wv}) \]

\[ > ( -2s_{nu} + s_{mu} + s_{nu+1} + a_{uw} s_{wv})^2. \quad (A.10) \]

Recall \( j^* = n \), then \( v < u \). Proceeding by contradiction. Suppose \( s_{nu+1} u + s_{nu} u + s_{nu+1} v + a_{uw} s_{wv} \)

Based on \( v < u \) and property (2) of Lemma 3.5 it follows that \( |s_{v+1} - e| < r^m s_{uw} \) and

\[ s_{nu} < r^{2(n-u+1)}(u - v) s_{uw} = r^n s_{uw}. \quad (A.11) \]

Replacing \( a_{uw} \) by \( -a_{uw} \) in (A.10), we obtain a new inequality. Adding this new inequality and (A.10) yields

\[ 4(s_{v+1} - s_{e})(s_{n+1} - s_{m}) > a_{uw} s_{wv}^2. \quad (A.12) \]

From \( |s_{v-1} u'| < r^m s_{uw}, |s_{v+1} v| < r^m s_{uw} \) and (A.12), it is seen that

\[ 8r^{2n-u-v} s_{uw} s_{wv} = 8r^n s_{uw} s_{wv}^2 > a_{uw} s_{wv}^2. \]

\[ \text{i.e., } 8r^{2m-u-v} s_{uw} > a_{uw} s_{wv}^2. \quad (A.13) \]

Since \( a_{uw} \) may be arbitrarily large, when \( a_{uw} \) is large enough, from (30) we get \( s_{uw} > r^n s_{uw} \) which contradicts (A.11). Thus we can conclude that \( s_{nu+1} u + s_{nu} u + s_{nu+1} v + a_{uw} s_{wv} \) and consequently \( s_{nu+1} u + s_{nu} u + s_{nu+1} v + a_{uw} s_{wv} \)

which is a contradictory requirement. Then suppose \( s_{uu} \leq r^n s_{uu} \). It follows from property (2) of Lemma 3.5 that \( |s_{v+1} | < r^{n-1} s_{uu} \) and

\[ s_{uu} < r^{n-1} r^{n-2} \cdots r^{n-(n-1)} s_{uu} = r^n s_{uu}. \quad (A.17) \]

On the other hand, set \( a_{n+1} s_{u+1} = 1 \) in (A.14), in light of \( s_{uu} > r^n s_{uu} \), (A.17) and (A.14), we may deduce

\[ 8r^n s_{uu} s_{uu} 4(s_{v+1} - s_{e})(s_{n+1} - s_{k}) > a_{uu} s_{u+1} s_{v+1} \]

\[ > a_{uu} s_{uu} s_{uu} \text{ i.e., } 16r^n m s_{uu} > a_{uu} s_{uu} s_{uu}. \quad (A.18) \]

when \( r \) is large enough. If \( a_{uu} \) is large enough, from (A.18) we can get \( s_{uu} > r^n s_{uu} \), which contradicts (A.17). Hence we conclude that the requirement of \( s_{uu} > r^n s_{uu} \) always lead to a contradiction.

When \( s_{uu} \leq r^{n-k+2} s_{kk} \). From (6) and property (2) of Lemma 3.5 it follows that \( |s_{v+1} | < r^{n-1} s_{uu} \) and

\[ s_{uu} < r^{n-k+2} s_{kk} \]

which is a contradictory requirement. Then suppose \( s_{uu} \leq r^n s_{uu} \). It follows from property (2) of Lemma 3.5 that \( |s_{v+1} | < r^{n-1} s_{uu} \) and

\[ s_{uu} < r^{n-1} r^{n-2} \cdots r^{n-(n-1)} s_{uu} = r^n s_{uu}. \quad (A.17) \]

On the other hand, set \( a_{n+1} s_{u+1} = 1/r^2 \) in (A.14), in light of \( s_{uu} > r^n s_{uu} \), (A.17) and (A.14), we see

\[ 8r^n s_{uu} s_{uu} 4(s_{v+1} - s_{e})(s_{n+1} - s_{k}) > a_{uu} s_{u+1} s_{v+1} \]

\[ > a_{uu} s_{uu} s_{uu} \text{ i.e., } 16r^n m s_{uu} > a_{uu} s_{uu} s_{uu}. \quad (A.18) \]

when \( r \) is large enough. If \( a_{uu} \) is large enough, from (A.18) we can get \( s_{uu} > r^n s_{uu} \), which contradicts (A.17). Hence we conclude that the requirement of \( s_{uu} > r^n s_{uu} \) always lead to a contradiction.

When \( s_{uu} \leq r^{n-k+2} s_{kk} \). From (6) and property (2) of Lemma 3.5 it follows that \( |s_{v+1} | < r^{n-1} s_{uu} \) and

\[ s_{uu} < r^{n-k+2} s_{kk} \]

which is a contradictory requirement. Then suppose \( s_{uu} \leq r^n s_{uu} \). It follows from property (2) of Lemma 3.5 that \( |s_{v+1} | < r^{n-1} s_{uu} \) and

\[ s_{uu} < r^{n-1} r^{n-2} \cdots r^{n-(n-1)} s_{uu} = r^n s_{uu}. \quad (A.17) \]

On the other hand, set \( a_{n+1} s_{u+1} = 1/r^2 \) in (A.14), in light of \( s_{uu} > r^n s_{uu} \), (A.17) and (A.14), we see

\[ 8r^n s_{uu} s_{uu} 4(s_{v+1} - s_{e})(s_{n+1} - s_{k}) > a_{uu} s_{u+1} s_{v+1} \]

\[ > a_{uu} s_{uu} s_{uu} \text{ i.e., } 16r^n m s_{uu} > a_{uu} s_{uu} s_{uu}. \quad (A.18) \]

when \( r \) is large enough. If \( a_{uu} \) is large enough, from (A.18) we can get \( s_{uu} > r^n s_{uu} \), which contradicts (A.19). Therefore, when \( 1 \leq k < n-1 \), we can always arrive at a contradiction.

Case ii: \( k = n-1 \). Since \( a_{n-1} s_{u+1} \) is a structural independent uncertainty, from Lemma 3.7 we have

\[ r^2 s_{n+1} n+1 > s_{nn} \text{ and } r s_{n+1} n+1 > |s_{n-1} n|. \quad (A.21) \]
From property (2) of Lemma 3.5 it follows that

\[ |s_{21}| < r^{n-1}s_{11} \]

and

\[ s_{n-1n-1} < r^2 \times r^2 \times \cdots \times r^2 \times (n-1) s_{11} \triangleq r^n s_{11}. \]

(A.22)

The requirement of \( \det(\pi(1: n - 1) > 0 \) leads to

\[ 4(s_{21} - s_{11})(s_{n - 1} - s_{n - 1} - s_{n - 1} - a_{n - 1}^2 s_{n}) > a^2_{n - 1} a^2_{n - 1} s_{11}. \]

(A.23)

Set \( a_{n - 1} + 1 = 1 \) in (A.23) and meanwhile note \( |s_{21}| < r^{n-1}s_{11} \) and (A.21). From (A.23) we may obtain

\[ 8r^{n-1}s_{11} \times 2r^2 s_{n - 1} - 1 > a^2_{n - 1} s_{11}^2 \]

i.e.,

\[ 16r^{n-1}s_{n - 1} - 1 > a^2_{n - 1} s_{11} \]

(A.24)

when \( r \) is large enough. If \( a_{n - 1}^2 \) is large enough, from (A.24) we have \( s_{n - 1} - 1 > r^n s_{11} \), contradicting (A.22).

References


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