Multi-objective optimal fixture layout design

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Abstract

This paper addresses a major issue in fixture layout design to determine and evaluate the acceptable fixture designs based on multiple quality criteria and to select an optimal fixture with appropriate trade-offs among multiple performance requirements. The performance objectives considered are related to the fundamental requirements of kinematic localization and total fixturing (form-closure). Three performance objectives are defined as the workpiece localization accuracy and the norm and dispersion of the locator contact forces. The paper focuses on multi-criteria optimal design with a hierarchical approach. An efficient interchange algorithm is extended and used for different practical cases, leading to proper trade-off strategies for performing fixture synthesis. Examples are given to illustrate empirical observations with respect to the proposed approach and its effectiveness.

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1. Introduction

Proper fixture design is crucial to product quality in terms of precision and accuracy in part fabrication and assembly. Fixturing systems, usually consisting of clamps and locators, must be capable of performing, positioning and holding the workpiece throughout a machining operation. Although there are a few design guidelines such as the 3-2-1 rule, automated systems for designing fixtures based on CAD models have been slow to evolve.

This article describes a research approach to automated design of a class of fixtures for 3D workpieces. The workpiece to be fixtured is of an arbitrary complex geometry and the fixture is considered to have the minimum number of elements required, namely, six locators and a clamp. Furthermore, the fixture elements are modeled as non-frictional point contacts and are restricted to be applied within a given collection of discrete candidate locations. In general, the set of fixture locations available is assumed to be a potentially very large collection. For example, the locations might be generated by discretizing the exterior surfaces of the workpiece. The goal of the fixture design is firstly to determine the feasible fixture configurations that satisfy deterministic localization and form-closure requirements. Secondly, the sets of acceptable fixture designs are evaluated based on several performance criteria and an optimal (or sub-optimal) fixture is selected. The performance measures considered in this work are localization accuracy and the norm and dispersion of the locator contact forces. These objectives cover the most critical considerations of a fixture design. These multiple objectives may involve a conflict. Thus, a trade-off has to be used in multi-objective fixture design.

An optimal fixture design method is presented in the paper. The approach is based on a concept of optimum experiment design. An algorithm is developed for efficient evaluation of the admissible designs exploiting the recursive properties in a localization and force analysis. The algorithm produces the optimal fixture design that meets a set of multiple performance requirements.

2. Related work

Literature on general fixturing techniques is substantial [1]. The essential requirement of fixturing is the century-old concept of form closure [2], which has been extensively studied in the field of robotics in recent years...
There are several formal methods for analyzing performance of a given fixture based on the popular screw theory, dealing with issues such as kinematic closure [5], contact types and friction effects [6]. A different analysis approach based on the geometric perturbation technique was reported in [7]. An automatic modular fixture design procedure based on this method was developed in [8] to include geometric access constraints in addition to kinematic closure. The problem of designing modular fixtures gained more attention lately [9]. There has also been extensive research on fixture designs, focusing on workpiece and fixture structural rigidity [6], tool accessibility and path clearance [7].

The problem of fixture synthesis has been largely studied for the case of a fixed number of fixture elements [8,10,11], particularly in the application to robotic manipulation and grasping for its obvious reasons [3,4]. The approach described in this article is an extension of the technique of fixture design previously reported in [14].

3. Fixture model

The fundamental performance of a fixture is characterized by the kinematic constraints imposed on the workpiece being held by the fixture. The kinematic conditions are well understood [3–5,7,12]. For a fixture with a given set of performance requirements.

\[ d_{\text{y}} = G^T \delta q, \]  

where \( d_{\text{y}} = [d_{\text{y}1}, d_{\text{y}2}, \ldots, d_{\text{yn}}]^T \) and \( \delta q = [\delta r^T, \delta \theta^T]^T \) define small perturbations in the locator positions and the location of the workpiece, respectively. The fixture design is defined by the locator matrix \( G = [h_1, h_2, \ldots, h_n] \) where \( h_i = [n_i^T, (r_i \times n_i)^T]^T \) and \( n_i \) and \( r_i \) denote the surface normal and position at the \( i \)th contact point on the workpiece surface. The problem of fixture design requires the synthesis of a fixturing scheme to meet a given set of performance requirements.

4. Quality performance criteria for a fixture

4.1. Accurate localization

An essential aspect of fixture quality is to position the workpiece into the fixture with high positioning precision. In general the workpiece positional errors are due to the geometric variability of the part and the locators set-up errors. This paper focuses only on the workpiece positional errors due to the locator positioning errors.

As an extension of the fixture model (Eq. (1)), the locator positioning errors \( d_{\text{y}} \) can be related with the workpiece localization error \( \delta q \) as follows:

\[ \|d_{\text{y}}\|^2 = \delta q^T (GG^T) \delta q. \]  

Clearly, for given source errors the workpiece positional accuracy depends only on the locator locations and is independent of the fixture clamp. The positioning errors are characterized by the system matrix, or Fisher information matrix, \( M = GG^T \). It has been shown [12] that a suitable criterion to achieve high-localization accuracy is to maximize the determinant of the information matrix (D-optimality), i.e., \( \max(\det M) \).

4.2. Minimal locator contact forces

Another objective in fixture layout design is to minimize support forces of the locator contacts due to any external force and clamping for complete kinematic restraint or force-closure. Let us consider the clamp location completely defined by the position vector \( r_c \) and unit surface normal \( n_c \). For the point contact, the generalized clamping force applied on the workpiece is given as

\[ F_c = h_c \lambda_c, \]

where \( \lambda_c > 0 \) represents the clamping force magnitude. Thus, the locator contact forces in response to the clamping action are given as

\[ \alpha_c = -G^T (GG^T)^{-1} h_c \lambda_c = -G^T M^{-1} h_c \lambda_c. \]

Normalizing these forces with respect to the clamping intensity we obtain

\[ p_c = \alpha_c / \lambda_c = -h_c^T M^{-1} h_c, \]

where \( p_{ci} = -h_i^T M^{-1} h_i (i = 1, 2, \ldots, n) \).

The force-closure condition requires these forces to be always positive for each locator \( i \) of a set of \( n \) locators

\[ p_{ci} > 0 \quad \text{for} \quad \lambda_c > 0. \]

Computing the norm of the locator contact forces

\[ \|p_c\|^2 = \sum_{i=1}^{n} p_{ci}^2 = h_c^T M^{-1} h_c, \]

leads to an appropriate design objective \( \min(\|p_c\|) \). Note that this objective indicates both locator and clamp positions should be determined in an optimal design process.

4.3. Balanced locator contact forces

Another significant issue in designing a fixture is that the total force acting on the workpiece should be distributed as uniformly as possible among the locator contacts. If \( \bar{\rho} \) represents the mean reactive force in
response to the clamp action, then we define the dispersion of the locator contact forces as

\[ d = \frac{1}{n} \sum_{i=1}^{n} (p_{ci} - \bar{p})^2, \quad \text{where} \quad \bar{p} = \frac{1}{n} \sum_{i=1}^{n} p_{ci}. \quad (7) \]

Therefore, minimizing the defined dispersion represents an objective for a balanced force-closure, \( \min(d) \).

5. Optimal fixture design with interchange algorithms

As mentioned earlier, by generating a set of discrete locations on the exterior surface of the workpiece we create a potential collection for the fixture elements. For example, using a CAD model of the workpiece, a discrete collection of unit normal vectors can be generated as uniformly as possible on the surfaces as candidates of fixture components, as illustrated in Fig. 1.

In the fixture layout design optimal candidates will be selected from this collection for locators and clamps with respect to the performance objectives and the kinematic closure condition. It is a complex task to deal with a large number of candidate locations in selecting an appropriate set of locators and a clamp.

As already introduced in [12,14] an effective method for finding the desired fixture with regard to one of the quality objectives defined in Section 4 is the optimal pursuit method with an interchange algorithm. As illustrated in the flowchart of Fig. 2, the algorithm consists of a sequence of three phases:

**Phase 1: Random generation of initial sets of locators.** An initial layout is generated by a random selection of a locator set consisting of six locators out of the list of \( N \) candidate locations. If the clamp is pre-determined, a valid locator set is obtained if the kinematic constraints are satisfied. Multiple initial sets of locators are repeatedly generated, such that each initial set will go through a further optimization process to obtain the best solution.

**Phase 2: Improvement by interchange.** The goal of interchange is to pursue for an improvement of an initial set of locators with respect to an objective. Basically, this is done iteratively by exchanging one by one the current locators with candidate locations from the global collection. It is also essential to consider the form-closure restraint during the exchange procedure. The process will continue as long as an improvement of the objective function is obtained. Based on some algebraic properties, an efficient evaluation for the best pair to be selected can be obtained for the objective functions \( \max(\det M) \), \( \min \Delta p_c \), and \( \min(d) \), respectively [12–14].

**Phase 3: Selecting the optimal solution.** Applying the interchange algorithm for each randomly generated initial set of locators, we will end up with several distinct solutions for an optimal fixture configuration scheme. The best fixture design corresponds evidently to improved layout with the best objective function value. It should be emphasized that this algorithm can be used sequentially for different objective functions. Depending on the objective pursued at the time, the best solution can be directly obtained for a single objective or may need the designer’s final decision for a balance among multiple objectives.

6. Multi-objective fixture locator optimization

In many applications the clamp could be pre-determined given some practical considerations. Then
with the clamp pre-defined, the best fixture with respect to a certain performance criterion can be constructed by selecting a suitable set of locators such that a significant improvement of the objective function is achieved.

Using the random interchange algorithm described above, we can analyze the impact of the optimization process on the fixture characteristics and determine the best fixture solution for a specific criterion. Extensive results using the algorithm for optimal fixture design with a single objective have been reported in [12,14]. Based on a set of statistical and empirical observations, it was concluded that three optimal performance measures max(det $M$), min $\Delta p_c$, and min($d$) usually have a conflict. Thus, a comprehensive fixture design requires a sensible balance among the multiple performance requirements. This is the focus of the paper and shall be discussed further below.

6.1. Multi-objective trade-offs

In some applications both localization quality and minimum force dispersion are important. In this case we may have to use a two-step algorithm: firstly max(det $M$) and secondly min($d$). Our numerical experience indicates that when maximizing the determinant it is likely that the force dispersion will decrease. On the other hand, a decrease in dispersion often leads to a decrease in the determinant value (det $M$) [14]. Thus, it is proposed to make the trade-offs between the two objectives in the specific order of optimization. For the multi-objective optimization problem the interchange algorithm is applied successively for both objectives. With the clamp pre-defined, the condition of form-closure is evaluated and maintained for each exchange step.

Numerical results for the multi-objective optimization of localization precision and the uniform contact force for the example of Fig.1 are presented in Figs. 3–6. Figs. 3 and 4 illustrate the changes of the fixture characteristics during the two-step algorithm performed on an initial random generation of locator sets with the clamp pre-fixed. The advantage of using max(det $M$) as the first step is noticed. While the determinant is increased, the norm and the dispersion of the contact forces are decreased, resulting a higher overall quality of the fixture. In the first step, the multiple initial locator sets are reduced to a smaller number of sub-optimal solutions owning to the interchange improvement. On the other hand, in the second phase when applying min($d$) it is observed that the determinant would decrease. This indicates a conflict between these two objectives, resulting in a trade-off in the final design solution. This fact is often expressed by the Pareto-line plot (Fig. 4). In this case the final decision has to be left for the designer to determine the best fixture scheme.

As an example, a single initial set of locators is studied for the interchange process of the two-step algorithm (Fig. 5) to further illustrate the observations. The trade-off zone is decisive in the multi-objective design. The resultant configurations of the fixture after each successive phase are presented in Fig. 6. It should be noted that the first phase of optimization for max(det $M$) objective moves the locators close to the surface boundaries of the part and away from each other as far as possible. On the other hand, the second phase
of optimization for $\min(d)$ objective results in a re-
positioning of the locators toward the interior of the
surfaces.

6.2. Design decision in finalizing the fixture

During the second phase of the algorithm a fairly
significant amount of decrease in the determinant value
may occur. This may render the solution unacceptable.
In order to prevent this problem, an interactive control
by the designer during $\min(d)$ interchange process is
recommended. Essentially, the interchange process has
to be controlled such that the determinant $(\det M)$ of the
improved locators must be maintained above a certain
bound. Even considering a tight bound for the
determinant, more solutions are often acceptable for
the design than in the uncontrolled min(d) optimization case. This is clearly illustrated in Fig. 7.

As an example, a single set of locators is studied during the interchange process with two different bounds controlling the determinant, respectively (Fig. 8). The corresponding final fixture configurations are also shown in the figure. This example indicates that the interactive control can play a decisive role in finalizing the best fixture configuration amid the performance trade-offs.

7. Optimal fixture clamping

This section deals with a more complicated problem to search simultaneously for the optimal clamp and locators in order to achieve a required fixture performance measure. By varying the clamp, it is obvious that the number of combinations for possible clamp-locators candidates would increase significantly. It is shown that this problem is manageable for the precision localization objective. For the other objectives we will have to restrain the search of the optimal clamp inside of a small set of locations, such that the optimization procedure could be handled efficiently.

7.1. Optimal clamp from a set of clamps

In some applications the clamps have certain preferred locations. Thus, there is a need to choose the best clamp location from a proposed collection. For example, let us consider that a collection of preferred clamps is given and an optimal fixture design with respect to the precision localization objective is needed. It is obvious that by applying a random interchange procedure successively for each clamp we will be able to find an optimal fixture configuration for each specified clamp. Among these fixture schemes, we will end up with an optimal clamp and its corresponding best locator set, constituting an optimal fixture design. Figs. 9 and 10 show such an example.
7.2. Optimal clamp from a set of clamps

Further, the approach to optimal clamp design can be extended to the multi-objective design problem. It consists of mainly applying the two-step interchange algorithm consecutively for each proposed clamp.

By collecting the results after applying this procedure for all the clamps, we can compare their results and select the most appropriate clamp. As described above, when considering the multi-objective trade-offs, each clamp may yield different result. This is shown in Fig. 11 for the example shown in Fig. 10. The best clamp should allow for a balance between the localization precision and the force dispersion as suggested in Fig. 11. The final fixture design with corresponding optimal locators is shown in Fig. 12, where the proposed clamp locations are shown on the left.

8. Conclusions

This article deals with optimal design of fixture layout for 3D workpieces with interchange algorithms. The optimization objectives considered include accurate workpiece localization, minimal and balanced contact forces. The paper focuses on multi-criteria optimal design with a hierarchical approach. The optimization processes make use of an efficient interchange process to achieve a trade-off among the specified objectives. Examples are used to illustrate empirical observations with respect to the design procedures and their effectiveness.

The work described here is yet complete. Since the inter-relationship between the locators and the clamps has a determinant role on the fixture quality measures, a more coherent and complete approach to study the influence of the clamp and to search for the optimal clamp position is needed in future works.

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References


Fig. 11. Clamp selection for multi-objective design.

Fig. 12. The initial collection of proposed clamps (left) and the best clamp and the corresponding optimal locators (right).