Stress-related topology optimization via level set approach

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1. Abstract

Considering stress-related objective or constraint functions in topology optimization problems is very important from both theoretic and application perspectives. It has been known, however, that this kind of problem is very challenging since several difficulties must be overcome in order to solve it efficiently. Traditionally, SIMP model is often adopted to tackle it. Although remarkable achievements have been obtained, there are still some issues that have not been well addressed within this computational framework. Stemming from this background, in the present work, stress-related topology optimization problems were investigated intensively via level set based optimization approach. It was pointed out that some of the difficulties encountered in the process of optimization employing traditional methods are due to the ill-posedness of the problem formulations. In order to cure the pathological phenomena, in the present paper, regularized formulations for stress-related topology optimization problems are proposed and the corresponding numerical solution aspects are discussed. Numerical examples show the effectiveness of the proposed approach and its capability to resolve the difficulties that cannot be dealt with conveniently via traditional approaches.

2. Keywords: shape optimization, topology optimization, stress constraints, level set, X-FEM

3. Introduction

Considering stress-related objective or constraint functions in topology optimization problems is very important from both theoretic and application perspectives. Topology optimization for continuum structures considering stress constraints has received more and more attentions in recent years [1-5].

4. Problem statement

Traditionally, stress-related topology optimizations are always formulated in the following forms:

\[
\begin{align*}
\text{find } & \chi(x) \in L^\infty(D), u(x) \in H^1(D) \\
\text{Minimize } & V = \int_D \chi(x) dV \\
\text{s. t. } & \int_D \chi(x) E_{ijkl} u_{i,j} v_{k,l} dV = \int_D f_i v_i dV + \int_{\Gamma_t} t_i v_i dS, \quad \forall v \in U_{ad} \\
& F(\chi(x), \sigma(u)) = \int_D \chi(x) \Gamma \left( \sigma(u(x)) \right) dV - \bar{F} \leq 0, \\
& u = \bar{u}, \quad \text{on } \Gamma_u, \\
& \chi(x) \in \{0,1\}, \quad \forall x \in D,
\end{align*}
\]

(1)
or

\[
\text{find } \chi(x) \in L^\infty(D), \ u(x) \in H^1(D) \\
\text{Minimize } F(\chi(x), \sigma(u)) = \int_D \chi(x) r\left(\sigma(u(x))\right) dV \\
\text{s.t. } \int_D \chi(x) E_{ijkl} u_{ij} v_{kl} dV = \int_D f_i v_i dV + \int_{\Gamma_t} t_i v_i dS, \quad \forall \nu \in U_{ad}, \\
\int_D \chi(x) dV - \bar{V} \leq 0, \\
\ u = \bar{u}, \text{ on } \Gamma_u, \\
\chi(x) \in \{0,1\}, \quad \forall x \in D. \quad (2)
\]

In Eq. (1) and Eq. (2), D is a prescribed design domain in which the topology of the structure can be varied. \(\chi(x)\) and \(u(x)\) are the characteristic function denoting the material distribution in D and the displacement field of the corresponding boundary value problem, respectively. \(v(x)\) is the test function while \(U_{ad} = \{v(x) | v(x) \in H^1(D), \ v(x) = 0 \text{ on } \Gamma_u\}\) is the admissible set it belongs to. \(f\) and \(t\) denote the body force density in D and the traction force on Neumann boundary \(\Gamma_t\), respectively, with \(\bar{u}\) denoting prescribed displacement on Dirichlet boundary \(\Gamma_u\). \(E = (E_{ijkl}) = E/(1+v)[I + v/(1-2v)\delta \otimes \delta]\) (\(I\) and \(\delta\) denoting the fourth and second order identity tensor, respectively) is the fourth order elasticity tensor of isotropic material. \(E\) and \(v\) are the Young’s modulus and Poisson ratio, respectively. \(F(\chi(x), \sigma(u))\) is a topology-dependent stress-related quantity which is generally a functional of the stress tensor and \(\bar{F}\) is its upper bound. In the present study, \(\Gamma_u\) and the part of \(\Gamma_t\) where \(t \neq 0\) are assumed to be fixed in the space for the sake of simplicity.

If level set framework is adopted to solve the problem, the corresponding optimization problems can be reformulated as follows:

\[
\text{find } \phi(x) \in L^\infty(D), \ u(x) \in H^1(D) \\
\text{Minimize } \ V = \int_D H(\phi(x)) dV \\
\text{s.t. } \int_D H(\phi(x)) E_{ijkl} u_{ij} v_{kl} dV = \int_D H(\phi(x)) f_i v_i dV + \int_{\Gamma_t} t_i v_i dS, \quad \forall \nu \in U_{ad}, \\
F(\phi(x), \sigma(u)) = \int_D H(\phi(x)) r\left(\sigma(u(x))\right) dV - \bar{F} \leq 0, \\
\ u = \bar{u}, \text{ on } \Gamma_u \\
\phi(x) \in \{0,1\}, \quad \forall x \in D. \quad (3)
\]

and

\[
\text{find } \phi(x) \in L^\infty(D), \ u(x) \in H^1(D) \\
\text{Minimize } F(\phi(x), \sigma(u)) = \int_D H(\phi(x)) r\left(\sigma(u(x))\right) dV \\
\text{s.t. } \int_D H(\phi(x)) E_{ijkl} u_{ij} v_{kl} dV = \int_D H(\phi(x)) f_i v_i dV + \int_{\Gamma_t} t_i v_i dS, \quad \forall \nu \in U_{ad}, \\
\int_D H(\phi(x)) dV \leq \bar{V}, \\
\ u = \bar{u}, \text{ on } \Gamma_u. \quad (4)
\]

In Eq. (3) and Eq. (4), \(\phi(x)\) is the so-called topology and/or shape description function which describes the material distribution in the design domain in an implicit way and \(H(x)\) is the well-known Heaviside function. The advantage of the level set based optimization framework is that it can account for the shape and topology changes of the structure simultaneously in a flexible way.

5. Difficulties associated with the stress-related topology optimization problems
It is well-known that topology optimization under stress constraints will suffer from the so-called singularity phenomenon. If traditional formulation is used to solve this kind of problem, sometimes the true optimal solution cannot be obtained via ground structure approach because of the discontinuity of the stress constraint function at the critical values of the topology design variables [6]. In order to deal with the stress singularity phenomenon, ε-relaxed approach proposed by Cheng and Guo [7] and its variants are often employed circumvent this problem [5]. These methods are all developed based on the ground structure model and Solid Isotropic Material with Penalization (SIMP) framework. In the present work, we will discuss how to deal with this challenging problem via level set framework.

6. Regularized formulations for stress-related topology optimization problems

As pointed out in the previous section, traditional formulation for topology optimization problem involving stress measures often suffer from the aforementioned unpleasant ill-posed behavior resulting from the conditional (on-off switch) properties of topology-dependent quantities. In order to overcome this difficulty and stabilize the optimization process, according to different design purpose, we propose to regularize the stress-related topology optimization problems in the following ways, i.e.,

\[
\text{find } \phi(x) \in L^\infty(D), u(x) \in H^1(D) \\
\text{Minimize } \quad V = \int_D H(\phi(x))dV \\
\text{s.t. } \int_D H(\phi(x))E_{ijkl}u_{ij}u_{kj}dV = \int_D H(\phi(x))f_jv_jdV + \int_{\Gamma_t} t_jv_jdS, \quad \forall \nu \in U_{ad}, \\
F(\phi(x), \sigma(u)) = \int_D H(\phi(x))r(\sigma(u(x)))dV - \bar{F} \leq 0, \\
U = \int_D H(\phi(x))E_{ijkl}u_{ij}u_{kj}dV \leq \bar{U}, \\
u = \bar{u}_i, \quad \text{on } \Gamma_u
\]

and

\[
\text{find } \phi(x) \in L^\infty(D), u(x) \in H^1(D) \\
\text{Minimize } \quad F(\phi(x), \sigma(u)) = \int_D H(\phi(x))r(\sigma(u(x)))dV \\
\text{s.t. } \int_D H(\phi(x))E_{ijkl}u_{ij}u_{kj}dV = \int_D H(\phi(x))f_jv_jdV + \int_{\Gamma_t} t_jv_jdS, \quad \forall \nu \in U_{ad}, \\
V = \int_D H(\phi(x))dV - \bar{V} \leq 0, \\
U = \int_D H(\phi(x))E_{ijkl}u_{ij}u_{kj}dV \leq \bar{U}, \\
u = \bar{u}_i, \quad \text{on } \Gamma_u
\]

respectively. In Eq. (5) and Eq. (6), \(U\) represents the elastic strain energy of the structure and \(\bar{U}\) is its properly prescribed upper bound. Another kind of regularized formulation can be written as

\[
\text{find } \phi(x) \in L^\infty(D), u(x) \in H^1(D) \\
\text{Minimize } \quad U = \int_D H(\phi(x))E_{ijkl}u_{ij}u_{kj}dV \\
\text{s.t. } \int_D H(\phi(x))E_{ijkl}u_{ij}u_{kj}dV = \int_D H(\phi(x))f_jv_jdV + \int_{\Gamma_t} t_jv_jdS, \quad \forall \nu \in U_{ad}, \\
F(\phi(x), \sigma(u)) = \int_D H(\phi(x))r(\sigma(u(x)))dV - \bar{F} \leq 0,
\]

where...
This can be viewed as the “dual” formulation of Eq. (6).

The considered problems can also formulated using only \( \phi = \phi(x) \) as unknown function. Under this circumstance, the corresponding optimization problem, for example (7), can be formulated as

\[
\begin{align*}
\text{find} & \quad \phi(x) \in L^\infty(D) \\
\text{Minimize} & \quad U = \int_D H(\phi(x))E_{ijkl}u_j^\phi u_k^\phi dV \\
F_\phi(\sigma(u^\phi)) & = \int_D H(\phi(x))r(\sigma(u(x))) dV - \bar{F} \leq 0, \\
V & = \int_D H(\phi(x))dV - \bar{V} \leq 0.
\end{align*}
\]

In Eq. (8), \( u^\phi = u^\phi(x) \) is the displacement field corresponding to \( \phi = \phi(x) \), which can be obtained implicitly by solving a boundary value problem.

The main advantage of the above regularized formulations consists in the fact that stiffness constraints has been introduced into the original ill-posed formulation explicitly, which can therefore plays the role of regularization by excluding the zero or small stiffness structures from the feasible design domain. Furthermore, using elastic strain energy, which is the direct measure of the compliance of the structure, as the objective function in (7) also has the advantage that it can help to eliminate the isolated small islands resulting from the coalescence of approaching holes during the process of optimization effectively. This is due to the fact that isolated small islands have no contribution to the overall stiffness of the structure but will occupy a certain amount of volume, which is obviously uneconomic from optimization point of view.

The effectiveness of the proposed regularized formulation in SIMP framework has been reported in [8].

7. Numerical solution aspects

In this section, some issues related to the numerical implementation of the proposed approach will be discussed in details.

7.1. Finite element analysis

The numerical method we used in this work for obtaining the structural response is the so-called extended finite element method (X-FEM) proposed in [Wei and Wang], which is a generalized version of the conventional XFEM approaches in the literature. Compared with the traditional ersatz material approach, the accuracy for computing the stress-related quantities can be greatly enhanced with the use of this approach. Moreover, in the present computation framework, a fixed uniform four-node rectangular finite element mesh can be used throughout the whole process of optimization without the need of re-meshing. The key points for this approach consistent in approximating the displacement field in form of

\[
\begin{align*}
\mathbf{u}^h(x) & = \sum_{i \in I} N_i(x)H(\phi(x))\mathbf{u}^h_i,
\end{align*}
\]

and using boundary element subdivision procedures for numerical integration. For more detailed discussions about this X-FEM approach, we refer the readers to [9].

7.2. Shape sensitivity analysis

In level-set based topology optimization approach, shape derivative is required for driving the evolution of the level set function \( \phi(x) \). For a general objective or constraint functional defined as

\[
\begin{align*}
\vartheta_\phi = \int_{D(\phi)} r(u^\phi) dV = \int_D H(\phi(x))r(u^\phi) dV,
\end{align*}
\]
if it is assumed that \( r(\mathbf{u}^\phi) \) is a differentiable function of \( \mathbf{u}^\phi \) (the displacement field corresponding to \( \phi \)), the body force is neglected (i.e., \( f = 0 \)). \( t \) is spatial fixed (i.e., \( \partial t / \partial t = 0 \), where \( t \) is a virtual time parameter used for tracing the evolution of \( \phi \)) and \( \mathbf{u} = 0 \) on \( \Gamma_u \), then it can be shown that the shape derivative corresponding to the boundary velocity field \( \mathbf{V} = \mathbf{V}(x) \) can be obtained as

\[
\mathbf{d}_\phi = \frac{\partial \phi}{\partial t} = \int_{\phi=0} \left( -E_{ijkl}u_{ij}w_{klj} + \kappa (\mathbf{t} \cdot \mathbf{w}) + (\mathbf{n} \cdot \nabla (\mathbf{t} \cdot \mathbf{w})) + r(\mathbf{u}^\phi) \right) (\mathbf{V} \cdot \mathbf{n}) \, dS,
\]

where \( \mathbf{n} = \nabla \phi / |\nabla \phi| \) and \( \kappa = \nabla \cdot (\nabla \phi / |\nabla \phi|) \) are the unit outward normal vector and the curvature (curvature in 2D and twice of the mean curvature in 3D case) of the boundary where \( \phi(x) = 0 \), respectively. If only traction free Neumann boundary can be varied, which is the assumed case in the present study, Eq. (11) can be further simplified into the following form

\[
\mathbf{d}_\phi = \frac{\partial \phi}{\partial t} = \int_{\phi=0} \left( -E_{ijkl}u_{ij}w_{klj} + r(\mathbf{u}^\phi) \right) (\mathbf{V} \cdot \mathbf{n}) \, dS,
\]

Function \( \mathbf{w} \) in Eq. (11) is the so-called adjoint displacement field, which is the solution of the following boundary value problem

\[
\begin{align*}
\text{find} & & \mathbf{w}(x) \in \mathbf{U}_{ad} \\
\int_{D} H(\phi)E_{ijkl}w_{ij}v_{kl} \, dV &= \int_{D} H(\phi) \frac{\partial r(u)}{\partial u_i} v_i \, dV, & \forall \mathbf{v}(x) \in \mathbf{U}_{ad}.
\end{align*}
\]

Specifically, if \( r(\mathbf{u}) = \mathbf{r}(\mathbf{u}(x)) \), then the weak form equilibrium equation in (13) should be written as

\[
\int_{D} H(\phi)E_{ijkl}w_{ij}v_{kl} \, dV &= \int_{D} H(\phi) \frac{\partial \mathbf{r}(\mathbf{u})}{\partial \sigma_{ij}} \sigma_{ij}(\mathbf{v}) \, dV, & \forall \mathbf{v}(x) \in \mathbf{U}_{ad},
\]

where \( \mathbf{e} = (\epsilon_{ij}) = \text{sym} \nabla \mathbf{u} \) is the infinitesimal strain tensor and \( \sigma_{ij} = E_{ijkl}\epsilon_{kl} \). It is also worth noting that if the local stress measure at point \( x = x_0 \) is considered, then the corresponding objective or constraint functional can be written as

\[
\tau_{\phi} = \int_{D} \delta(x-x_0)H(\phi(x))\mathbf{r}(\mathbf{u}(x)) \, dV,
\]

where \( \delta = \delta(x) \) is the Dirac function. Noting that \( \tau_{\phi} \) is non-zero only for \( x \in D \) such that \( H(\phi(x)) = 1 \), which means that local stress constraint is only meaningful for the points occupied by solid materials.

Moreover, if the considered objective/constraint function is defined on some parts of the boundary of the structure, for example,

\[
\gamma_{\phi} = \int_{S} h(\mathbf{u}(x)) \, dS,
\]

then the corresponding shape derivative can be calculated as

\[
\mathbf{d}_\phi = \frac{\partial \phi}{\partial t} = \int_{S} (E_{ijkl}u_{ij}w_{klj} - \kappa (\mathbf{t} \cdot \mathbf{w}) - \mathbf{n} \cdot \nabla (\mathbf{t} \cdot \mathbf{w}) + \nabla \cdot \mathbf{n} + \kappa h) (\mathbf{V} \cdot \mathbf{n}) \, dS,
\]

where the adjoint displacement field \( \mathbf{w} \) is the solution of the following boundary value problem

\[
\begin{align*}
\text{find} & & \mathbf{w}(x) \in \mathbf{U}_{ad} \\
\int_{D} H(\phi)E_{ijkl}w_{ij}v_{kl} \, dV &= - \int_{S} \frac{\partial h(\mathbf{u}(x))}{\partial \sigma_{ij}} \sigma_{ij}(\mathbf{v}) \, dS, & \forall \mathbf{v}(x) \in \mathbf{U}_{ad}.
\end{align*}
\]

In the present study, finite element method was used to obtain the approximate adjoint displacement field \( \mathbf{w}^h \). The corresponding discrete equilibrium equations for \( \mathbf{w}^h \) can be written as

\[
\mathbf{K}(\phi)\mathbf{w}^h = \mathbf{f}_{ad}^h,
\]

where

\[
\mathbf{K}(\phi) = \sum_{e} \mathbf{K}^e = \sum_{e} \int_{V^e} \mathbf{B}^T\mathbf{DB} \, dV^e,
\]
\[ f^h_{ad} = \sum_e f^h_{ae} = \sum_e \int_{\Omega_e} B^T D m dV^e \]  
\[ m = \left( \frac{\partial r}{\partial \sigma_{ij}} \right)^T. \]

In the above equations, \( B \) and \( D \) denote the strain and elasticity matrix in finite element analysis, respectively. \( m \) is a column vector with appropriate dimension. Furthermore, it is also worth noting that according to whether the global or local objective/constraint functional is considered, \( f^h_{ad} \) will correspond to a set of distribution force or an concentrated force applying at the point where the local structural response is concerned, respectively.

The shape derivative of the volume functional is relatively simple, which can be derived as
\[ \frac{dV}{dt} = \frac{d}{dt} \int_{\Omega} H(\phi(x)) dV = \int_{\phi=0} (V \cdot n) dS. \]  

7.3. Optimization algorithm

For the considered problem, augmented Lagrangian multiplier method is used to solve the optimization problem with multiple inequality constraints. To this end, an augmented Lagrangian functional should be first established. For optimization problem of (8), the following augmented functional can be constructed
\[ L = U + \lambda_1 \left( F_{\phi} \left( \sigma(u^\phi) \right) - \bar{F} \right) + \lambda_2 (V - \bar{V}), \]
\[ \min_{\phi} \max_{\lambda_1, \lambda_2 \geq 0} L = \min_{\phi} \max_{\lambda_1, \lambda_2 \geq 0} \left( U + \lambda_1 \left( F_{\phi} \left( \sigma(u^\phi) \right) - \bar{F} \right) + \lambda_2 (V - \bar{V}) \right). \]

For fixed values of \( \lambda_1 \geq 0 \) and \( \lambda_2 \geq 0 \), the shape derivative of \( L \) can be expressed as
\[ \frac{dL}{dt} = \int_{\phi=0} \left( -E_{ijkl} u_{ij}^\phi u_{kl}^\phi + \lambda_1 \left( -E_{ijkl} u_{ij}^\phi u_{kl}^\phi + r(u^\phi) \right) + \lambda_2 \right) (V \cdot n) dS, \]
where \( w \) is the aforementioned adjoint displacement field. If the normal velocity component \( V_n = V \cdot n \) along the boundary for the evolution of \( \phi \) is taken as
\[ V_n = - \left( -E_{ijkl} u_{ij}^\phi u_{kl}^\phi + \lambda_1 \left( -E_{ijkl} u_{ij}^\phi u_{kl}^\phi + r(u^\phi) \right) + \lambda_2 \right), \]
then we have
\[ \frac{dL}{dt} = - \int_{\phi=0} \left( -E_{ijkl} u_{ij}^\phi u_{kl}^\phi + \lambda_1 \left( -E_{ijkl} u_{ij}^\phi u_{kl}^\phi + r(u^\phi) \right) + \lambda_2 \right)^2 dS \leq 0, \]
which ensures the descent of \( L \) at every evolution step of \( \phi \). The Lagrange multipliers can be adjusted in the following iterative way
\[ \lambda_1^{k+1} = \max \left( \lambda_1^k + \frac{1}{\mu} \left( \int_{\Omega} H(\phi^k) dV / \bar{V} - 1 \right), 0 \right), \]
\[ \lambda_2^{k+1} = \max \left( \lambda_2^k + \frac{1}{\mu} \left( F_{\phi} \left( \sigma(u^\phi) \right) / \bar{F} - 1 \right), 0 \right), \]
where \( \mu \) is a penalty parameter.

7.4. Evolution of the level set function

The level set function will be evolved according to the following Hamilton-Jacobi function
\[ \frac{\partial \phi}{\partial t} + V_n |\nabla \phi| = 0. \]
In the present paper, upwind scheme was used for the discretization of spatial derivatives and forward Euler algorithm was used to integrate, i.e.,

$$\Phi_{ij}^{k+1} = \Phi_{ij}^{k} + \Delta t \left[ \max(V_{ij}^n, 0) \nabla^+ + \min(V_{ij}^n, 0) \nabla^- \right]$$  \hspace{1cm} (32)

where

$$\nabla^+ = \left[ \max(D_{ij}^{-x}, 0)^2 + \max(D_{ij}^{+x}, 0)^2 + \max(D_{ij}^{-y}, 0)^2 + \max(D_{ij}^{+y}, 0)^2 \right]^{\frac{1}{2}},$$  \hspace{1cm} (33)

$$\nabla^- = \left[ \min(D_{ij}^{-x}, 0)^2 + \max(D_{ij}^{+x}, 0)^2 + \min(D_{ij}^{-y}, 0)^2 + \max(D_{ij}^{+y}, 0)^2 \right]^{\frac{1}{2}}.$$  \hspace{1cm} (34)

In the above equations, $$V_{ij}^n$$ and $$\Phi_{ij}^k$$ denote normal component of the velocity field and the value of $$\Phi$$ at $$(i,j)$$ node of the finite element mesh at $$k$$-th time step. $$D_{ij}^{-x}$$ and $$D_{ij}^{+y}$$ are the forward and backward finite difference operators along coordinate directions, respectively.

$$\Delta t$$ is the time step for temporal integration which should satisfies the CFL stability condition. It is worth noting that if $$\theta$$ is a domain integral, then the boundary velocity in Eq. (27) can be extended to the entire region of D in a natural way. Furthermore, in order to ensure the numerical stability, the level set function should be re-initialized into a signed distance function after several iteration steps.

7.5. Treatment of local stress constraint

In the present study, two types of stress-related measure are considered. One is the integral of the Von Mises stress on the whole structure, i.e.

$$F_1 = \int_{\Omega} H(\Phi(x))\bar{f}\left(\sigma(u(x))\right) dV,$$

where $$\bar{f}\left(\sigma(u(x))\right) = \sqrt{\sigma_{xx}^2(u) + \sigma_{yy}^2(u) - \sigma_{xx}\sigma_{yy}(u) + 3\sigma_{xy}^2(u) \geq 0}$$. The other one is the point-wise maximum value of the Von Mises stress in the structure, i.e.

$$F_2 = \max_{x_0 \in \Omega} \int_{\Omega} \delta(x - x_0)H(\Phi(x))\bar{f}\left(\sigma(u(x))\right) dV.$$

If $$F_2$$ is involved in the optimization problem, traditionally, Kreisselmeier-Steinhauser (K-S) function or big $$p$$-norm method is often used to transform it into an approximate smooth functional, which is a global integral on the whole structure, for numerical implementation purpose. In the present work, the point-wise Von Mises stress constraint was dealt with in a direct way, that is, at every iteration step, we pick out the maximum point-wise value of $$F_2$$ from all Gauss-points in the structure and then use the shape derivative with respect to the local stress constraint on this selected Gauss point to drive the next step evolution of $$\Phi$$. The effectiveness of this approach has been demonstrated by numerous numerical examples.

8. Numerical examples

In this section we will present a numerical example to illustrate the effectiveness of the proposed approach. The dimensionless Young’s moduli are set to 1 and $$1 \times 10^{-6}$$ for the solid material and weak material, respectively. The Poisson’s ratio is $$\nu = 0.3$$ for both solid and week material.

The classical L shape beam is shown in Figure 1. The finite element mesh resolution is 80x80. A force $$F = 1$$ is applied at the middle point of the right side. The upper bound of the available material ratio is 0.2.
For this example, we intent to minimize the compliance of the structure while restricting the maximum local Mises stress of the structure less than 70. The optimal structures and the corresponding stress contour are shown in Figure 2. Table 1 gives the comparison of the optimal results for different cases considered. It can be seen from the optimization results that when local stress constraint is considered, the re-enter corner of the optimal structure becomes more smoother than that obtained without considering local stress constraint. This is quite reasonable from mechanics point of view.

Figure 1: The initial design of the L shape beam

Figure 2a: Optimal structure without stress constraint

Figure 2b: Optimal structure with max $\bar{f}(\sigma) \leq 70$. 
9. Concluding remarks
In the present study, a level set based computational framework has been proposed for stress-related topology optimization problem. The attractive aspects of this computational framework can be summarized as follows: (1) Shape and topology change of the structure can be dealt with in a flexible way via the level set function, (2) Since there is no grey region in the design domain therefore the subtle issue of defining the allowable stress as in the SIMP model can be circumvented, (3) The proposed regularized formulation can help to eliminate the pathological phenomena which may deteriorate the process of optimization significantly, (4) The accuracy of the stress computation can be enhanced with X-FEM method without resorting to the time-consuming refining of mesh adaptively.

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11. References


